

# Formulaire : trigonométrie circulaire et hyperbolique

Propriétés trigonométriques : remplacer cos par ch et sin par i · sh.

$$\cos^2 x + \sin^2 x = 1$$

$$\cos(a + b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

$$\sin(a + b) = \sin a \cdot \cos b + \sin b \cdot \cos a$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$$

$$\cos(a - b) = \cos a \cdot \cos b + \sin a \cdot \sin b$$

$$\sin(a - b) = \sin a \cdot \cos b - \sin b \cdot \cos a$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$$

$$\begin{aligned} \cos 2a &= 2 \cos^2 a - 1 \\ &= 1 - 2 \sin^2 a \\ &= \cos^2 a - \sin^2 a \end{aligned}$$

$$\sin 2a = 2 \sin a \cdot \cos a$$

$$\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$$

$$\cos a \cdot \cos b = \frac{1}{2} [\cos(a + b) + \cos(a - b)]$$

$$\sin a \cdot \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$$

$$\sin a \cdot \cos b = \frac{1}{2} [\sin(a + b) + \sin(a - b)]$$

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cdot \cos \frac{p-q}{2}$$

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \cdot \sin \frac{p-q}{2}$$

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cdot \cos \frac{p-q}{2}$$

$$\sin p - \sin q = 2 \sin \frac{p-q}{2} \cdot \cos \frac{p+q}{2}$$

Avec  $t = \tan \frac{x}{2}$  on a

$$\begin{cases} \cos x &= \frac{1-t^2}{1+t^2} \\ \sin x &= \frac{2t}{1+t^2} \\ \tan x &= \frac{2t}{1-t^2} \end{cases}$$

$$\cos' x = -\sin x$$

$$\sin' x = \cos x$$

$$\tan' x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\arccos' x = \frac{-1}{\sqrt{1-x^2}} \quad (|x| < 1)$$

$$\arcsin' x = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$$

$$\arctan' x = \frac{1}{1+x^2}$$

$$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$$

$$\operatorname{ch}(a + b) = \operatorname{ch} a \cdot \operatorname{ch} b + \operatorname{sh} a \cdot \operatorname{sh} b$$

$$\operatorname{sh}(a + b) = \operatorname{sh} a \cdot \operatorname{ch} b + \operatorname{sh} b \cdot \operatorname{ch} a$$

$$\operatorname{th}(a + b) = \frac{\operatorname{th} a + \operatorname{th} b}{1 + \operatorname{th} a \cdot \operatorname{th} b}$$

$$\operatorname{ch}(a - b) = \operatorname{ch} a \cdot \operatorname{ch} b - \operatorname{sh} a \cdot \operatorname{sh} b$$

$$\operatorname{sh}(a - b) = \operatorname{sh} a \cdot \operatorname{ch} b - \operatorname{sh} b \cdot \operatorname{ch} a$$

$$\operatorname{th}(a - b) = \frac{\operatorname{th} a - \operatorname{th} b}{1 - \operatorname{th} a \cdot \operatorname{th} b}$$

$$\begin{aligned} \operatorname{ch} 2a &= 2 \operatorname{ch}^2 a - 1 \\ &= 1 + 2 \operatorname{sh}^2 a \\ &= \operatorname{ch}^2 a + \operatorname{sh}^2 a \end{aligned}$$

$$\operatorname{sh} 2a = 2 \operatorname{sh} a \cdot \operatorname{ch} a$$

$$\operatorname{th} 2a = \frac{2 \operatorname{th} a}{1 + \operatorname{th}^2 a}$$

$$\operatorname{ch} a \cdot \operatorname{ch} b = \frac{1}{2} [\operatorname{ch}(a + b) + \operatorname{ch}(a - b)]$$

$$\operatorname{sh} a \cdot \operatorname{sh} b = \frac{1}{2} [\operatorname{ch}(a + b) - \operatorname{ch}(a - b)]$$

$$\operatorname{sh} a \cdot \operatorname{ch} b = \frac{1}{2} [\operatorname{sh}(a + b) + \operatorname{sh}(a - b)]$$

$$\operatorname{ch} p + \operatorname{ch} q = 2 \operatorname{ch} \frac{p+q}{2} \cdot \operatorname{ch} \frac{p-q}{2}$$

$$\operatorname{ch} p - \operatorname{ch} q = 2 \operatorname{sh} \frac{p+q}{2} \cdot \operatorname{sh} \frac{p-q}{2}$$

$$\operatorname{sh} p + \operatorname{sh} q = 2 \operatorname{sh} \frac{p+q}{2} \cdot \operatorname{ch} \frac{p-q}{2}$$

$$\operatorname{sh} p - \operatorname{sh} q = 2 \operatorname{sh} \frac{p-q}{2} \cdot \operatorname{ch} \frac{p+q}{2}$$

Avec  $t = \operatorname{th} \frac{x}{2}$  on a

$$\begin{cases} \operatorname{ch} x &= \frac{1+t^2}{1-t^2} \\ \operatorname{sh} x &= \frac{2t}{1-t^2} \\ \operatorname{th} x &= \frac{2t}{1+t^2} \end{cases}$$

Dérivées : la multiplication par i n'est plus valable

$$\operatorname{ch}' x = \operatorname{sh} x$$

$$\operatorname{sh}' x = \operatorname{ch} x$$

$$\operatorname{th}' x = 1 - \operatorname{th}^2 x = \frac{1}{\operatorname{ch}^2 x}$$

$$\operatorname{Argch}' x = \frac{1}{\sqrt{x^2 - 1}} \quad (x > 1)$$

$$\operatorname{Argsh}' x = \frac{1}{\sqrt{x^2 + 1}}$$

$$\operatorname{Argth}' x = \frac{1}{1-x^2} \quad (|x| < 1)$$